

Why π is Irrational

$$\pi = \sum_{i=1}^{\infty} \left(\frac{4}{4i-3} - \frac{4}{4i-1} \right) = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \dots$$

The decimal representation of every fraction must terminate or repeat within fewer digits than the value of its denominator. The denominators of the above sequence grow large without bound. Then the number of repeating digits in the decimal fractions derived from that sequence is also unbounded. The number of repeating digits in the sum of those decimal fractions is the least common multiple of the number of repeating digits in each nonterminating decimal in the sequence. The least common multiple of this sequence increases without bound. The number of digits that occur in π before the pattern repeats is unbounded. Therefore, the digits of π do not repeat.

For example: For terms 4 and 9 with denominators of 7 and 17, the number of repeating digits is 6 and 16, respectively. The LCM of 6 and 16 is 48. So, after the ninth term, the approximation repeats every 48 digits.

The tenth denominator is 19 with 18 repeating digits. The LCM changes to 144.

The twelfth denominator is 23 with 22 repeating digits. The LCM becomes 1,584.

The fifteenth denominator is 29 with 28 repeating digits. The LCM is 11,088.

The sixteenth denominator is 31 with 30 repeating digits. The LCM is 55,440.

The number of repeating digits is far greater than the accuracy of the approximation provided by each additional term.